2014-2015 Nova Scotia Assessment: Mathematics in Grade 6

Lessons Learned

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NOVA SCOTIA

Before we begin…

If you have any questions during the presentation, please post them in the chat at the left of your screen, and we will spend some time during the webinar as a Q/A time.

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Description

The purpose of this webinar is to introduce teachers, school administrators, board mathematics leads and others to the Lessons Learned document for M6 and how to use it.

The document is intended as a support for all elementary teachers, in particular grades 4 – 6 teachers. There are five Lessons Learned in this document. Each lesson is supported by the assessment results, along with pedagogy; next steps in instruction for a class or individual students, and appropriate methods and activities for assessing student learning.

The document is a wonderful support for professional development

Welcome

- Introductions
- Logistics

Goals of the Webinar

- Background on the creation of the Lessons Learned Document
- Format of the Lessons Learned Document
- Data contained within it
  - Problem Solving
  - Patterns and Relations
  - Number
  - Measurement
  - Estimation

Cognitive Levels - Knowledge Questions (Level 1)

Knowledge questions (Level 1) may require students to recall or recognize information, names, definitions, or steps in a procedure.

Example: For each fraction, write the equivalent decimal.

a. $\frac{25}{100}$ is the same as _____

b. $\frac{6}{10}$ is the same as _____

c. $\frac{9}{100}$ is the same as _____

Cognitive Levels – Application Questions (Level 2)

Application/comprehension questions (Level 2) may require students to make connections, represent a situation in more than one way (translating between representations), or solve contextual problems.

Example:

Elastics are sold in boxes of 500 elastics, large bags of 100 elastics, small bags of 50 elastics, balls of 10 elastics, and single elastics. What would you buy if you wanted 753 elastics?

Cognitive Levels - Analysis Questions (Level 3)

Analysis questions (Level 3) may require students to go beyond comprehension and application to higher order thinking skills, such as generalizations and non-routine problem-solving.

Example:

Melinda wants to build a pen for her chickens. She bought 6 large squares of plywood for the floor of her chicken pen. She arranges the squares so that sides are touching. Find all possible shapes for the floor of the pen. How much fencing would she need for each pen?

Lesson Learned 1 - Problem Solving

What conclusions can be drawn from the assessment data?

Students have

- good understanding of Level 1 questions
- basic procedural knowledge and facts under control
- can find a solution when explicitly given all the information needed to do a Level 1 (knowledge) question

Problem Solving

What conclusions can be drawn from the assessment data?

Areas of Challenge

- Problem solving is a challenge for students across all strands.
- Level 2 and Level 3 questions are problematic for most students.
- Students are not able to apply higher order thinking skills.
- Many students cannot translate between representations of a concept (i.e. words to pictures)

Problem Solving

EXAMPLE: (Possible Strategy: Use a model; Strand: Number; Level 3)

Antoine bought 32 batteries. He put 8 batteries in his remote control model car. His 3 sisters divided the rest of the batteries equally. How many batteries did each sister receive?

Solution:

- Use counters to represent the batteries (32)
- Remove 8 counters (representing the 8 batteries used in remote control car) from the 32 counters
- 24 batteries are left to divide equally among his 3 sisters
- Take the 24 counters. Divide the 24 counters into 3 piles (each pile represents one of the sisters; each sister gets 8 batteries)
- Could you solve this problem another way?

Problem Solving

Do students have any misconceptions or errors in their thinking?

- Many students have the misconception that the “word problem” (solving a problem in a context) is always too hard for them to attempt.
- Most students tend not to have a strategy to begin the problem.
- Putting words around numbers obstructs their ability to think about the question.
- Students believe there is only one way to solve a problem.

Problem Solving

Do students have any misconceptions or errors in their thinking?

- Students do not understand how to represent their thinking with words, pictures, and/or manipulatives when asked to solve a word problem.
- Most students only provide symbols when solving a contextual problem.
- Many students find it difficult to find an entry point to begin to solve a problem.

Problem Solving

Entry Points to Solve a Problem

Kelly has a quilt that has 24 squares. One-fourth of the squares are yellow, $\frac{3}{6}$ are green, and the rest are red. What colour is the greatest number of squares?

Solution 1: Use equivalent fraction knowledge

Solution 2: Use the information given (draw a picture)

Entry Points to Solve a Problem

Kelly has a quilt that has 24 squares. One-fourth of the squares are yellow, $\frac{3}{6}$ are green, and the rest are red. What colour is the greatest number of squares?

Solution 1: Use equivalent fraction knowledge

Green. Since three-sixths is the same as one-half, we know that green has the greatest number of squares. There are two other colours of squares, so each of them must be less than one-half the quilt.

$$\frac{3}{6} = \frac{1}{2}$$

Entry Points to Solve a Problem

Kelly has a quilt that has 24 squares. One-fourth of the squares are yellow, $\frac{3}{6}$ are green, and the rest are red. What colour is the greatest number of squares?

Solution 2: Use the information given (draw a picture)

Green has the greatest number of squares.

Green	Green	Red	Yellow
Green	Green	Red	Yellow
Green	Green	Red	Yellow
Green	Green	Red	Yellow
Green	Green	Red	Yellow
Green	Green	Red	Yellow

Problem Solving

What are the next steps in instruction for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- learn mathematics as a result of problem solving
- to think about the problem and work through the solution in a variety of ways, and only then draw the procedures out of their work
- be exposed to problem-solving strategies (personal or explicitly taught)
- encourage students to represent their thinking with words, pictures, and/or manipulatives when asked to solve a word problem

Problem Solving

From Reading Strategies to Mathematics Strategies

- With a problem-solving approach embedded and expected throughout our curriculum grades Primary to 12 in all strands, there are definite implications for the teaching of reading strategies in mathematics.

- When teachers use these strategies in the instructional process or as assessment tasks, the expectations for students must be made explicit.

- Please refer to the Appendix B found at the end of the document for further strategies with illustrative examples.

Problem Solving

What are the most appropriate methods and activities for assessing student learning?

- Encourage students to share their thinking, their strategies, their representations, and their solutions to problems.
- Make connections between the representations and solutions shared by students.
- Sample problems have been provided on pages 11-12.
- There are numerous examples of questions for problem solving in both the curriculum documents and the core resources.

Lesson Learned 2 - Patterns and Relations

What conclusions can be drawn from the assessment data?

Students

- have good understanding of Level 1 questions
- were able to extend a pattern to find the next term when given a pattern rule

(Level 2 question)

Patterns and Relations

What conclusions can be drawn from the assessment data?

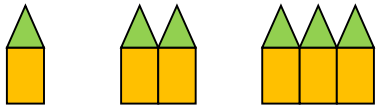
Areas of Challenge

- continuing and extending a pattern to predict a subsequent term that is not consecutive
- using patterns and relationships to solve problems
- representing a word problem with an equation containing an unknown

Patterns and Relations

EXAMPLE

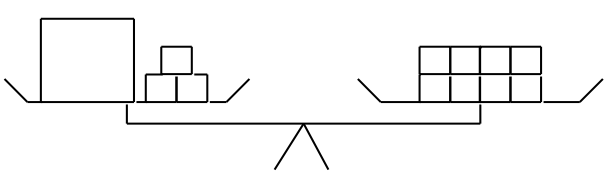
Look at the picture below. The first term is made up of two pattern blocks. The second term is made up of four pattern blocks, and the third term is made up of six pattern blocks. Predict the number of pattern blocks in the eighth term.

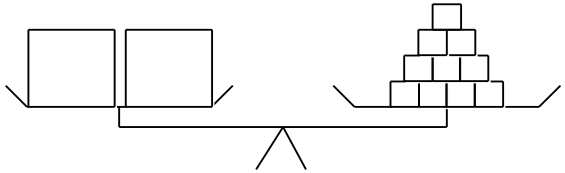


Patterns and Relations

EXAMPLE

Write equations to describe the balance representations, using letters for the variables.





Patterns and Relations

Do students have any misconceptions or errors in their thinking?

- Many students do not do not recognize that there are different ways to continue a pattern if a pattern rule is not given.

- If presented with a sequence such as 5, 10, 15 . . . . , students may only see it as a repeating pattern (5, 10, 15, 5, 10, 15, 5, 10, 15, …) and will not consider that it might be a growing [increasing] pattern, for example, 5, 10, 15, 20, 25, 30 … or 5, 10, 15, 25, 35, 50, 65, … or 5, 10, 15, 25, 40, 65, 105,…

Patterns and Relations

Do students have any misconceptions or errors in their thinking?

- At times, students omit important information in describing a pattern rule. For example: if describing the pattern 4, 6, 9, 13, 18 . . ., a student might state the rule as “Just keep adding 1 more.”

- A suggested strategy is to follow students’ rules literally to show them the incomplete nature of their rules. For example, with the rule “Just keep adding 1 more,” you might continue the pattern by writing down 4, 6, 9, 13, 18, 19, 20, 21 …literally adding 1 more.

- This should help the students see the need for a clearer and more complete rule such as, “Start with 4 and add 2, then add 3, then add 4, each time adding a number that is 1 greater than the time before.”

Patterns and Relations

What are the next steps in instruction for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- begin by representing a pattern with concrete materials and/or pictures
- use a variety of vocabulary to help describe the patterns that they find in charts and tables
- create two representations of the same pattern – one with materials or pictures, and one that is in a table or chart
- find and explain place-value patterns
- describe what is happening as the pattern increases (or decreases) and how the next step is related to the previous one

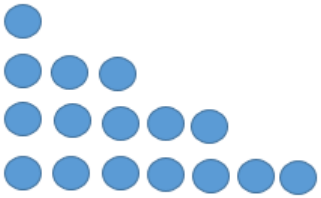
Patterns and Relations

Lennie was training for a marathon. On Day 1 of his training, he ran 1 km. On Day 2 of his training, he ran 3 km. On Day 3 of his training, he ran 5 km.

If he continues with this pattern for his training, on which day of his training will he run 15 km? Use words, pictures, and a table to explain your thinking.

Words: Start at 1. Add 2 each day. 1, 1 + 2, 1 + 2 + 2, 1 + 2 + 2 + 2, ….

Words: Double the term value, and subtract 1. (1 x 2) - 1



Day	1	2	3	4	5	6	7	8
Distance (km)	1	3	5	7	9	11	13	15

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Patterns and Relations

What are the most appropriate methods and activities for assessing student learning?

- Encourage students to share their thinking, their strategies, their representations, and their solutions.
- Make connections between the representations and solutions shared by students.
- Sample problems have been provided on pages 15-17.
- There are numerous examples of questions in both the curriculum documents and the core resources for patterns and relations.

Lesson Learned 3 - Number

What conclusions can be drawn from the assessment data?

Students

- have good understanding of Level 1 questions
- have basic procedural knowledge, skills, and facts under control
- are able to calculate sums, differences, products, and quotients
- understand the context of most problems
- are able to represent fractions
- are successful when given all the information need to solve a problem

Number

What conclusions can be drawn from the assessment data?

Areas of Challenge

- questions that require application of knowledge and analysis (Level 2 and Level 3)
- recognizing the operation represented by the story problem
- solving contextual story problems
- using strategies to solve problems
- equivalent fractions

Number

What conclusions can be drawn from the assessment data?

Areas of Challenge

- translating between representations of an operation
  - understanding the connection between words and symbols used to represent operations
  - explaining the connection between the models and the story problems using verbal expressions such as “groups of,” “rows of,” and “jumps of” for multiplication and division

- understanding or using the relationship between addition and subtraction and/or between multiplication and division

Whole Number: Addition and Subtraction Misconceptions and Errors

Students have the misconception that they always subtract the **smaller** digit from the larger digit regardless of the position of that digit, for example

451	(minuend)	5**0**9	6**23**
– 231	(subtrahend)	– 3**8**9	- **4**78
220	(difference)	2**8**0	255

Whole Number: Addition and Subtraction Misconceptions and Errors

Students forget to regroup when adding, for example

$$\begin{array}{r} 14\mathbf{5} \\ +\ 24\mathbf{7} \\ \hline 38\mathbf{12} \end{array}$$

Whole Number: Addition and Subtraction Misconceptions and Errors

Students misalign the digits when recording their calculations and compute incorrectly.

$$\begin{array}{r} 4\,5\,6\,7 \\ +\ 3\,7\,4 \\ \hline 8\,3\,0\,7 \end{array}$$

Whole Number: Multiplication Misconceptions and Errors

Students have a misconception about multiplication centered on place value.

For example, if given a multiplication question such as 23 X 41, some students read this and say, “4 times 2 is 8” and record an 8, when they should be saying, “4 tens times 2 tens is 800”.

Whole Number: Multiplication Misconceptions and Errors

Students ignore the zeros in a number.

For example, when calculating 504 X 2, students may simply multiply 2 x 4, and 2 x 5 and record each product, rather than multiplying 2 x 4 ones, 2 x 0 tens, and 2 x 5 hundreds.

504	504
X 2	X 2
108	1008

Whole Number: Division Misconceptions and Errors

This same misconception applies when dividing whole numbers containing dividends with zeros.

609 ÷ 3 = 23

840 ÷ 4 = 21

Decimals
Misconceptions and Errors

- Students do not believe that decimals are numbers.
- Students do not recognize the differences between decimal numbers when reading decimals numbers.
- Many students believe that when computing with decimals, you line up the digits regardless of the place value of those digits.
- Students do not understand that a whole number (without a defined decimal) can be represented with a decimal.

Decimals

For example, when adding 5.2 + 8.5 + 22, students do not know how to express 22 as a number with a decimal.

5.2	5.2
8.5	8.5
+ 2 2	+ 22.0
1 5.9	35.7

Fractions
Misconceptions and Errors

Students attempt to apply whole-number thinking to fractions. For example, many think that since seven is more than four, sevenths are larger than fourths.

Students do not realize that fractions are part of one whole (region, set, measure).

Students do not define the “whole” and assume that a fraction represents a specific quantity rather than a relationship between a part and the whole.

Whole Number Operations

What are the next steps in instruction for the class and for individual students?

Insist that students estimate before performing calculations. This will alert them to the reasonableness of their calculated answers.

For example, before calculating the product of 23 x 41, students should estimate the product as 20 x 40 = 800. If students estimate before attempting to calculate, they should realize that 25 is not a reasonable solution for 23 x 41, but 943 is a reasonable solution.

23	
X 41	
8	(4 x 2)
12	(4 x 3)
2	(1 x 2)
+ 3	(1 x 3)
25	

23	
X 41	
800	(40 x 20)
120	(40 x 3)
20	(1 x 20)
+ 3	(1 x 3)
943	

Whole Number Operations

What are the next steps in instruction about addition and subtraction for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- have many opportunities to solve and create word problems for the purpose of answering real-life questions, preferably choosing topics of interest to them

- solve story problems of all structures for addition and subtraction, and for multiplication and division

- start with a word problem and then use materials to model the problem and to determine the solution

Whole Number Operations

What are the next steps in instruction about addition and subtraction for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- write number sentences that reflect their thinking about story problems
- be able to model operations using base-ten blocks, and use symbols to record the processes that reflect their actions with those blocks
- draw pictures to represent **their** thinking and mirror their work with models

Whole Number Operations

What are the next steps in instruction about addition and subtraction for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- share their thinking, their strategies, their representations, and their solutions to problems

- perform calculations using “personal strategies” that are efficient, accurate, and mathematically correct

Whole Number Operations

What are the next steps in instruction about multiplication and division for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- recall basic facts quickly and accurately
- use a variety of concrete and pictorial models to investigate operations to help them develop an understanding of the connection between the models and the symbols
- use correct mathematical language as they manipulate the materials and pictorially record their work with base-ten blocks

Whole Number Operations

What are the next steps in instruction about multiplication and division for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- focus on the relationships between operations to ensure that they recognize that, for example, multiplication and division are two ways of looking at the same situation

- discuss the concept of remainders

Decimals

What are the next steps in instruction about decimals for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- represent decimal numbers with proportional concrete and pictorial models; these may include ten-frames, base-ten blocks, or grids
- represent decimal numbers using words, models, pictures, and symbols and make connections among various representations
- connect decimals to whole numbers and to fractions
- read decimals in context
- say decimals correctly as this will help students make the connection between decimals and fractions

Decimals

What are the next steps in instruction about decimals for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- develop strategies for adding and subtracting decimals using concrete materials, such as base-ten blocks and number lines
- be exposed to the variety of story problem structures so that they get a full picture of the various contexts in which decimals are used
- recognize that all of the properties and developed strategies for the addition and subtraction of whole numbers also apply to decimal numbers
- to recognize that **estimation** is a useful skill when doing computations

Fractions

What are the next steps in instruction about fractions for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- recognize that a fraction can name part of a set, part of one whole, part of a measure
- recognize that a fraction is a comparison of a part to the whole
- understand that the size of the whole can change
- visualize equivalent fractions as the naming of the same region or set partitioned in different ways

Fractions

What are the next steps in instruction about fractions for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- model fractions using many different concrete materials
- model fractions using number lines, areas, and sets
- explore fractions in meaningful contexts
- continue to use conceptual methods to compare fractions
- understand that fractions can only be compared if they are parts of the same whole

Number

What are the most appropriate methods and activities for assessing student learning?

- Encourage students to share their thinking, their strategies, their representations, and their solutions to problems.
- Make connections between the representations and solutions shared by students.
- Sample problems have been provided on pages 29-31.
- There are numerous examples of questions for number in both the curriculum documents and the core resources.

Lesson Learned 4 - Measurement

What conclusions can be drawn from the assessment data?

Students

- have good understanding of Level 1 questions
- have a good understanding of time
- can read time on an analog clock, and express it numerically
- are able to solve word problems involving elapsed time (L2)

Areas of Challenge

- perimeter and area when presented together in application (Level 2) and analysis questions (Level 3)
- measuring with a ruler

Measurement

Do students have any misconceptions or errors in their thinking?

- Sometimes students forget to include the measures of unlabeled sides when working with perimeter.
- Many students rely on numbers alone, without considering the units of the measures.
- Many students incorrectly place the ruler when measuring an object.
- Many students do not realize that the scale of the ruler begins at 0 cm, especially if it is not labelled at the beginning of the ruler.

Measurement

What are the next steps in instruction for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- understand concepts of perimeter and area presented in real-world problem-solving contexts
- learn about area and perimeter together
- recognize that area and perimeter are independent of one another

Measurement

What are the next steps in instruction for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- use geo-boards, grid paper, and tiles to construct rectangles having
  - different areas and perimeters
  - the same perimeter but different areas
  - the same area but different perimeters

- estimate measurements before actually verifying them using a measuring tool

Measurement

What are the most appropriate methods and activities for assessing student learning?

- Encourage students to share their thinking, their strategies, their representations, as they share solutions.
- Make connections between the representations and solutions shared by students.
- Sample problems have been provided on pages 34-35.
- There are numerous examples of questions in both the curriculum documents and the core resources for measurement.

Lesson Learned 5 - Estimation

What conclusions can be drawn from the assessment data?

Areas of Challenge

- Students do not like to use estimation strategies, unless specifically asked to do so in a question.

- Students appear not to use any strategy for estimation and simply guess at an answer.

- Many students just want to get the “right” answer and feel there is no value in estimating before calculating or measuring.

Estimation

Do students have any misconceptions or errors in their thinking?

- Students think that an estimate is a random guess.
- Some students believe that estimates are either right or wrong.
- Place value often plays a role in misconceptions and errors in student thinking.
- Students do not understand that rounding both factors up or down gives an estimate that is not as close as rounding one factor up and one factor down.
- When students were asked to estimate the area of a geometric figure, they appeared to lose track of their count when counting whole and part squares.

Estimation

What are the next steps in instruction for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- model estimation strategies
- solve a variety of estimation problems related to contexts that are meaningful to them
- transfer the use of operations and estimation strategies to situations found in their daily lives

Estimation

What are the next steps in instruction for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- find estimates, so they are alert to the reasonableness of pencil-and-paper or calculator answers
- use the language of estimation; are *about*, *just about*, *between*, *a little more than*, *a little less than*, *close*, *close to*, and *near*
- make use of place-value understandings in estimating or calculating
- talk about place-value concepts while explaining reasoning and estimates

Estimation

What are the most appropriate methods and activities for assessing student learning?

- Encourage students to share their thinking, their strategies, their representations, and their solutions.
- Make connections between the estimation strategies and solutions shared by students.
- Sample problems have been provided on page 39.
- There are numerous examples of questions in both the curriculum documents and the core resources for estimation.

Appendices

Appendices found in the end of the document:

- Appendix A: Cognitive Levels of Questioning
- Appendix B: From Reading Strategies to Mathematics Strategies
- Appendix C: NSA: Mathematics 6 Lessons Learned – Cognitive Levels of Sample Questions

Summary

The ***Lessons Learned*** document for Mathematics in Grade 6 is designed as individual lessons so that you may take Lesson 1, for example, out as a stand alone piece and use it to support a school goal or your classroom assessment practice.

We hope the ***Lessons Learned*** document for Mathematics in Grade 6 will provide support to PLCs, staff PD and board PD throughout the province for teachers in all grades.

Questions and Contact Information

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This session has been recorded and the archive will be available within the next two weeks on the DVL website.
http://dvl.ednet.ns.ca/browse/results/taxonomy%3A169

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