# Lessons Learned 2013-2014 Nova Scotia Mathematics 4 April 1, 2015 

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## Before we begin...

If you have any questions during the presentation, please post them in the chat at the left of your screen, and we will spend some time during the webinar as a $\mathrm{Q} / \mathrm{A}$ time.
This session will be recorded and the archive will be available within the next two weeks on the DVL website. http://dvl.ednet.ns.ca

Please download the 2013-2014 M4-LessonsLearned.pdf document. Click on "Documents to download" button from the bottom right of this screen.
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## Description

The purpose of this webinar is to introduce teachers, administrators, board mathematics leads, and others to the Lessons Learned document for M4 and how to use it.

The document is intended as a support for all elementary teachers, in particular grades Primary - Grade 3 teachers. There are three Lessons Learned in this document.

Each Lesson Learned will be discussed in relation to the assessment results, expected pedagogy, next steps in instruction for a class or individual students, and appropriate methods and activities for assessing student learning.
The document is a wonderful support for professional development.

## Welcome

- Introductions
- Logistics


## Goals of the Webinar

- Background on the creation of the Lessons Learned Document
- Format of the Lessons Learned Document
- Data contained within it
- Translating Between and Among Representations
- Representing and Partitioning Whole Numbers
- Whole Number Operations


## Cognitive Levels - Level 1

Knowledge questions (Level 1) may require students to recall or recognize information, names, definitions, or steps in a procedure.

Example:
For each set of numbers, circle the number that is greatest.

| a. | 433 | 157 | 86 | 100 |
| :--- | :---: | :---: | :---: | :---: |
| b. | 508 | 580 | 850 | 805 |
| c. | 925 | 936 | 919 | 931 |
| d. 800 | 888 | 808 | 880 |  |

## Cognitive Levels - Level 2

Application/comprehension questions (Level 2) may require students to make connections, represent a situation in more than one way (translating between representations), or solve contextual problems.

## Example:

Elastics are sold in boxes of 500 elastics, large bags of 100 elastics, small bags of 50 elastics, balls of 10 elastics, and single elastics.
What would you buy if you wanted 753 elastics?

## Cognitive Levels - Level 3

Analysis questions (Level 3) may require students to go beyond comprehension and application to higher order thinking skills, such as generalizing and non-routine problem-solving.

## Example:

Bill represented the number 634 with base-ten blocks. He hid some of them behind the screen as shown below. What base-ten blocks are hidden behind the screen?
Explain how you know.


## Lesson Learned 1 - Translating Between and Among Representations

What conclusions can be drawn from the assessment data?

Students:

- have good understanding of Level 1 questions
- have basic procedural knowledge and facts under control
- understand the context of the problems


## Translating Between and Among Representations

What conclusions can be drawn from the assessment data?

Areas of Challenge:

- Level 2 questions that require students to apply knowledge.
- Students want to rush to the symbolic.
- Many students cannot translate between representations of a concept (i.e. from words to pictures).


## Translating Between and Among Representations

## Example:

What operation is represented by the picture of the base-ten blocks?

Record the equation represented by the picture.

© Small cube represents 1

## Translating Between and Among Representations

Do students have any misconceptions or errors in their thinking?

- Students relied heavily on symbols to try to solve problems.
- They did not understand how to represent their thinking with words, pictures, and/or manipulatives.
- They could not translate between and among representations of a concept.


## Translating Between and Among Representations

## What are the next steps in instruction for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- use a variety of representations to communicate mathematical ideas
- select among representations to represent a concept
- translate among representations and make connections between them
- explain how a concept may be represented with contexts, models, pictures, words, and symbols


## Translating Between and Among Representations

The importance of making explicit the connections between representations cannot be overstated.


## Translating Between and Among Representations

## What are the most appropriate methods and activities for assessing student learning?

- Encourage students to share their thinking, their strategies, their representations, and their solutions to problems.
- Make connections between the representations shared by students.
- Sample problems have been provided on page 5.
- There are numerous examples of questions that require translating among and between representations in both the curriculum documents and the core resources.


## Lesson Learned 2- Representing and Partitioning Whole Numbers

What conclusions can be drawn from the assessment data?

Students

- have good understanding of Level 1 questions
- have basic procedural knowledge, skills, and facts under control
- understand the context of most problems
- can calculate sums and differences
- are successful when given all the information in a problem, including what to do. For example: The pattern rule is start at 2 and add 3 each time. What is the next term in the pattern? $2,5,8, \ldots$


## Representing and Partitioning Whole Numbers

What conclusions can be drawn from the assessment data?

Areas of Challenge:

- Students struggle to translate between representations of a concept (i.e. words to pictures).
- Students struggle to decompose and recompose numbers in two or more parts (partition).
- Students are not able to connect unconventional or non-conventional representations of numbers.


## Representing and Partitioning Whole Numbers

What conclusions can be drawn from the assessment data?

Areas of Challenge:

- Level 2 questions that require students to apply knowledge are problematic.
- Students have difficulty using strategies to solve problems that are more open-ended.
- Students cannot apply partitioning to help them solve problems.


## Representing and Partitioning Whole Numbers

## Example:

What number is represented by the base-ten blocks below?


## Representing and Partitioning Whole Numbers

## Examples:

What number is represented by each expression?

$$
\begin{aligned}
& 5+60+900 \\
& 100+200+70+4+10
\end{aligned}
$$

$\qquad$

Write the numeral that makes the sentence true.
The number 78 tens is the same as the number $\qquad$ .

The number 7 hundreds, 18 ones is the same as the number $\qquad$ .

## Representing and Partitioning Whole Numbers

## Do students have any misconceptions or errors in their thinking?

Small cube represents 1


Students

- do not know the base-ten blocks and don't know they can be used to represent numbers
- do not understand that the small cube represents one and so count all the blocks as one block and record the total number of blocks (10)
- use a different block to represent one (i.e. flat) (523)
- count the blocks from right to left and record a digit for each "count" (5122)


## Representing and Partitioning Whole Numbers

Do students have any misconceptions or errors in their thinking?

Small cube represents 1

Students


- may miscount the blocks (542)
- don't identify the place value represented by the various blocks; start with the ones, then the tens, ... and record digits (253)
- count each "kind" of block and simply record the digit for each count (532)
- understand the value of the blocks, but incorrectly record the numeral (300502)


## Representing and Partitioning Whole Numbers

What are the next steps in instruction for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- represent numbers with models, words, and symbols
- make connections between the various representations
- understand equal but different representation of a number


## Representing and Partitioning Whole Numbers

What are the next steps in instruction for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- partition numbers in two or more parts with models, words, and symbols
- view, represent, and partition numbers in conventional and unconventional displays using models, words, and symbols
- display part-part-whole thinking for all numbers


## Representing and Partitioning Whole Numbers

The importance of making explicit the connections between representations cannot be overstated.

Represent the number 235 in as many ways as possible.

$$
\begin{aligned}
& 200+30+5 \\
& 5+30+200 \\
& 5+5+25+50+50+100 \\
& 100+20+5+100+10
\end{aligned}
$$



## Legend

(0) represents 1

200
30
$\begin{array}{r}1 \\ +\quad 5 \\ \hline\end{array}$

## Representing and Partitioning Whole Numbers

What are the most appropriate methods and activities for assessing student learning?

- Encourage students to share their thinking, their strategies, their representations, and their partitions as they share solutions.
- Make connections between the representations and partitions shared by students.
- Sample problems have been provided on pages 7-8.
- There are numerous examples of questions that require students to represent and partition numbers in both the curriculum documents and the core resources.


## Lesson Learned 3 - Whole Number Operations

What conclusions can be drawn from the assessment data?

Students

- have good understanding of Level 1
- have basic procedural knowledge, skills, and facts under control
- understand the context of most problems
- can calculate sums and differences
- are successful when given all the information in a problem


## Whole Number Operations

What conclusions can be drawn from the assessment data?

Areas of Challenge

- Level 2 and Level 3 questions that require students to apply knowledge and perform analysis are problematic for them.
- Although able to calculate when presented with a question such as $24+78$, students did not know which operation to use to solve contextual story problems.
- Students did not understand or use the relationship between addition and subtraction.
- Students have difficulty using strategies to solve problems.


## Whole Number Operations

Do students have any errors or misconceptions about addition and/or subtraction?

- Typically, students used traditional algorithms. This caused them to focus on single digits within the computation, rather than thinking about the number as a whole.
- When using a traditional algorithm, they did not appear to use reasoning to determine if their solution was reasonable.


## Whole Number Operations

Students have the misconception that they always subtract the smaller number from the larger number regardless of the position of the number in the question.

Example

| 451 | (minuend) | 509 | 613 |
| ---: | :--- | ---: | :---: |
| -231 | (subtrahend) | -389 | $\frac{-497}{284}$ |

## Whole Number Operations

Students forget to regroup when adding, for example

$$
\begin{array}{r}
145 \\
+247 \\
\hline 3812
\end{array}
$$

## Whole Number Operations

Students misalign the digits when recording their calculations and compute incorrectly.

| 241 |
| ---: | ---: |
| +63 |
| 871 | | 79 |
| ---: |
| 29 |

## Whole Number Operations

Do students have any errors or misconceptions about multiplication and/or division?

Students

- although able to calculate when presented with a question such as 3 x 5 , did not know which operation to use to solve a contextual story problem
- did not understand or use the relationship between multiplication and division
- have difficulty when asked to explain the connection between their models and the story problems using verbal expressions such as "groups of," "rows of," and "jumps of" for multiplication


## Whole Number Operations

Do students have any errors or misconceptions about multiplication and/or division?

Students

- have the misconception that the product of two numbers is always greater than the sum of those two numbers. So, when they encountered expressions like $8 \times 1,8 \times 0$, and $2 \times 2$ where this did not apply, they were puzzled
- have difficulty when asked to explain the connection between words and symbols for division


## Whole Number Operations

What are the next steps in instruction about addition and subtraction for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- be exposed to and understand how to estimate sums and differences
- understand that addition and subtraction are related, they undo each other
- understand that basic facts of addition and subtraction do not have to be learned separately

$$
\begin{aligned}
& 6+5=11 \text { so }, 11-5=6, \text { and } 11-6=5 \\
& 11=6+5 \text { so, } 6=11-5 \text {, and } 5=11-6
\end{aligned}
$$

## Whole Number Operations

What are the next steps in instruction about addition and subtraction for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- conceptually understand the story problem structures
- connect the story structures to personally meaningful experiences
- Include story problems that represent
- join
- separate
- part-part-whole
- comparison situations
- for more information about the story structures and instructional strategies, please refer to the grade level appropriate curriculum documents (Mathematics 1, page 64; Mathematics 2, page 68; and Mathematics 3, page 71)


## Whole Number Operations

What are the next steps in instruction about addition and subtraction for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- use and describe strategies to determine sums and differences using manipulatives and visual aids. Initial strategies include
- counting on or counting backwards
- one more or one less
- making ten
- doubles
- near doubles


## Whole Number Operations

What are the next steps in instruction about addition and subtraction for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- use manipulatives to model the story structures
- Examples of manipulatives that can be used for this purpose include
- two-sided counters
- linking cubes
- game materials (number cubes)
- ten-frames
- walk-on number line


## Whole Number Operations

What are the next steps in instruction about multiplication for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- model multiplication situations (equal groups/sets, arrays, and linear measurement models, such as number lines)
- For example, $3 \times 5$ can be represented with the
- Equal Group/Set Model - represent 3 plates of 5 cookies by making 3 groups of 5 counters
- Array Model - represent 3 rows of 5 cadets on parade by making 3 rows of 5 counters
- Linear Model - represent 3 jumps of 5 on a number line


## Whole Number Operations

What are the next steps in instruction about multiplication for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- correctly interpret and create story problems
- model story problems pictorially
- write repeated addition number sentences to represent them
- refrain from using the word "times" and the multiplication symbol too early because this may interfere with students' understanding of multiplication situations
- understand and use correct mathematical terms when describing multiplication situations
- understand the different ways that factors and products can be represented
- represent their contextual multiplication problems using all five representations of a concept


## Whole Number Operations

What are the next steps in instruction about multiplication for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- be exposed to samples of multiplication situations (equal-group story problems) that lend themselves to modelling with sets, arrays, and linear measurement models
- Please see pages 12-13 on the document for sample questions


## Whole Number Operations

What are the next steps in instruction about division for the class and for individual students?

Provide learning and classroom assessment opportunities that require students to

- introduce division through story problems, using the two types of situations, equal-sharing and equal-grouping, which need to be considered
- solve types of division problems by modelling them concretely, recording them pictorially, and describing the division in words before they are introduced to division sentences
- Please see page 13 in the document for sample questions.


## Whole Number Operations

What are the most appropriate methods and activities for assessing student learning?

- Encourage students to share their thinking, their strategies, their representations, as they share solutions.
- Make connections between the representations and solutions shared by students.
- Sample problems have been provided on pages 14-16.
- There are numerous examples of questions that require students to work with addition, subtraction, and the introduction of multiplication and division of whole numbers in both the curriculum documents and the core resources.


## Summary

The Lessons Learned document for Mathematics in Grade 4 is designed as individual lessons so that you may take Lesson 1, for example, out as a stand alone piece and use it to support a school goal or your classroom assessment practice.

We hope the Lesson Learned document for Mathematics in Grade 4 will provide support to PLCs, staff PD and board PD throughout the province for teachers in all grades.

## Questions and Contact Information



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This session has been recorded and the archive will be available within the next two weeks on the DVL website. http://dvl.ednet.ns.ca/browse/results/taxonomy\%3A169

